Wavelength-selective optical waveguide couplers with three parallel waveguides

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Wavelength-selective optical waveguide couplers with three parallel waveguides

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Summary

For three parallel coupled waveguides coupled mode analysis yields relations for complete power conversion, coupling length and wavelength selectivity, and facilitates the design of λ-duplexers. The beam propagation method is employed to analyse wave transmission and conversion in the effective index distribution of three parallel diffused channel-guides. With a 1-dB coupling bandwidth of 40 to 80 nm at 1.5 μm wavelength such triple channel couplers need 1 to 4 mm coupling length to duplex optical carriers spaced by 100 to 200 nm.

Wellenlängenselektive Koppler aus drei parallelen optischen Wellenleitern

Zusammenfassung

Eine Analyse der Wellenkopplung in drei parallelen Wellenleitern ergibt Beziehungen für volle Leistungskonversion, Koppellänge und Wellenlängenselektivität, mit denen sich Wellenlängenduplexer bemessen lassen. Mit der Strahlausbreitungsmethode wird die Übertragung und Konversion in der effektiven Brechzahlverteilung von drei parallelen diffundierten Kanalwellenleitern untersucht. Bei einer 1-dB-Koppelbandbreite von 40 bis 80 nm für 1.5 μm Wellenlänge brauchen solche Drei-Kanal-Koppler 1 bis 4 mm Koppellänge, um optische Träger im Wellenlängenbestand von 100 bis 200 nm zu verzweigen.

1. Introduction

Optical waveguide directional couplers with a wavelength-selective over-all coupling may find application in optical communication systems that transmit several optical carriers in wavelength multiplex over the same monomode fibre. A typical case is the bi-directional transmission of optical signals for duplex operation over a single fibre. Signals in one direction are transmitted on one wavelength and in the opposite direction on another. Terminals and repeaters for such duplex operation can use wavelength-selective directional couplers as in Fig. 1 to transmit on one wavelength λ_1, and receive on the other wavelength λ_2. Because of the selectivity of the coupler nearly all light power from the transmitter at λ_1, will be launched into the fibre, while nearly all the power arriving through the fibre at λ_2 will be transferred to the receiver. Further-

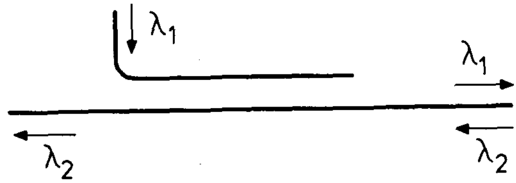

Fig. 1:
Wavelength-selective directional coupler as transmit receive duplexer

more the selectivity of the directional coupler enhanced by its directivity will almost completely suppress the near-end crosstalk. The lowest order fundamental mode of the fibre in which the signal arrives may have any polarization. It will always pass the wavelength-selective directional coupler with little conversion loss.

The triple-waveguide directional coupler in Fig. 2 works particularly well as such a duplexer [1]. At a wavelength where a mode of the center waveguide is phase-matched to the fundamental mode of the outer two guides all power can be converted between these outer guides, while for any substantial phase difference between the center and the outer waveguide modes the power is transmitted through the coupler without much conversion.

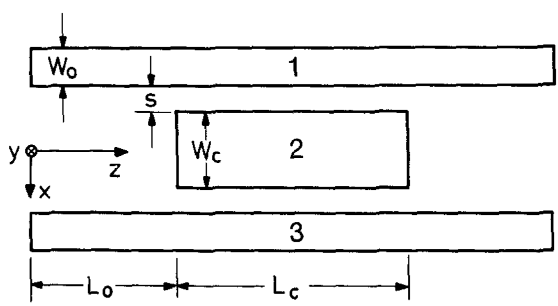

Fig. 2:
Triple-waveguide directional coupler

2. Coupled wave analysis

For a simple analysis of the triple-waveguide coupler it is assumed that a mode 1 with amplitude A_1 in waveguide 1 in Fig. 2 couples only to a mode 2 with amplitude A_2 in the center waveguide 2, and this mode 2 in turn couples only to a mode 3 with amplitude A_3 in waveguide 3. We let the amplitude coupling coefficient between mode 1 and 2 be equal to that between mode 2 and 3 and designate it by c. We furthermore let the phase coefficients of modes 1 and 3 be equal and designate them by β, while β_2 is the phase coefficient of mode 2 in the center guide. The mode amplitudes will then obey the following system of coupled wave equations.

$$\begin{aligned} dA_1/dz &= -j\beta A_1 - jcA_2 \\ dA_2/dz &= -jcA_1 - j\beta_2 A_2 - jcA_3 \\ dA_3/dz &= -jcA_2 - j\beta A_3. \end{aligned} \tag{1}$$

This system with three coupled waves has three normal modes which propagate independently from one another in the coupling region. Their phase coefficients are the eigenvalues of the matrix

$$-j\begin{bmatrix} \beta & c & 0 \\ c & \beta_2 & c \\ 0 & c & \beta \end{bmatrix} \tag{2}$$

of coefficients on the right hand sides of Eqs. (1) and are given by

$$\beta_{v-1} = \beta - \delta + \sqrt{\delta^2 + 2c^2};\ \ \beta_v = \beta;\ \ \beta_{v+1} = \beta - \delta - \sqrt{\delta^2 + 2c^2} \tag{3}$$

where

$$\delta = (\beta - \beta_2)/2 \tag{4}$$

designates half the difference in phase coefficients between mode 1 and 2, and mode 3. The eigenvector matrix of the coefficient matrix (2) of the coupled wave equations may be written as

$$\begin{bmatrix} 1 & -1 & 1 \\ \dfrac{\sqrt{\delta^2+2c^2}-\delta}{c} & 0 & -\dfrac{\sqrt{\delta^2+2c^2}+\delta}{c} \\ 1 & 1 & 1 \end{bmatrix} \tag{5}$$

so that the three normal modes with amplitudes w_{v-1}, w_v and w_{v+1} superimpose to form the following general solution for the amplitudes of the coupled modes 1, 2 and 3

$$\begin{aligned} A_1 &= w_{v-1}e^{-j\beta_{v-1}} - w_v e^{-j\beta_v} + w_{v+1}e^{-j\beta_{v+1}} \\ A_2 &= \frac{\sqrt{\delta^2+2c^2}-\delta}{c}\, w_{v-1}e^{-j\beta_{v-1}} - \frac{\sqrt{\delta^2+2c^2}+\delta}{c}\, w_{v+1}e^{-j\beta_{v+1}} \\ A_3 &= w_{v-1}e^{-j\beta_{v-1}} + w_v e^{-j\beta_v} + w_{v+1}e^{-j\beta_{v+1}}. \end{aligned} \tag{6}$$

With these wave amplitudes the normal modes $v \pm 1$ are even, while mode v is an odd mode.

If unit power is launched into mode 1 of waveguide 1 at $z=0$ we have as initial conditions

$$A_1 = 1 \qquad A_2 = A_3 = 0. \tag{7}$$

Under these conditions the normal modes are excited with the following amplitudes

$$w_{v-1} = \frac{1}{4} + \delta/(4\sqrt{\delta^2+2c^2});\ w_v = -\frac{1}{2};\ w_{v+1} = \frac{1}{4} - \delta/(4\sqrt{\delta^2+2c^2}). \tag{8}$$

Two limiting cases are of special interest:

1) $\beta_2 = \beta$: Mode 2 in the center guide is phase matched to modes 1 and 3 of the outer guides. In this case we have $\delta = 0$, the normal modes have the phase coefficients

$$\beta_{v\pm1} = \beta \pm \sqrt{2}c, \;\; \beta_v = \beta, \tag{9}$$

the eigenvector matrix reduces to

$$\begin{bmatrix} 1 & -1 & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}, \tag{10}$$

and for the initial conditions (7) the normal modes are exited with

$$w_{v\pm1} = \frac{1}{4} \pm \delta/(4\sqrt{\delta^2+2c^2}), \;\; w_v = -\frac{1}{2}\,. \tag{11}$$

The amplitudes of waves 1 and 3 amount for these initial conditions and $\delta=0$ to

$$\begin{aligned} |A_1| &= \frac{1}{2}\,|1+\cos(\sqrt{2}cz)| \\ |A_3| &= \frac{1}{2}\,|1-\cos(\sqrt{2}cz)|\,. \end{aligned} \tag{12}$$

The power which is launched into wave 1 at $z=0$ transfers completely to wave 3 at

$$z = (2q+1)\pi/(\sqrt{2}c) \quad \text{where } q = 0,1,2\ \ldots \tag{13}$$

For complete power conversion the coupler should best be

$$L_c = \pi/(\sqrt{2}c) \tag{14}$$

long.

2) $|\delta| \gg c$ is the other limiting case. In this case the normal modes have the phase coefficients

$$\beta_{v-1} \simeq \beta + c^2/\delta; \;\; \beta_v = \beta; \;\; \beta_{v+1} \simeq \beta - 2\delta \tag{15}$$

while the eigenvector matrix reduces approximately to

$$\begin{bmatrix} 1 & -1 & 1 \\ c/\delta & 0 & -2\delta/c \\ 1 & 1 & 1 \end{bmatrix}. \tag{16}$$

For initial conditions according to Eq. (7) the normal modes are in this limiting case exited with

$$w_{v-1} = \frac{1}{2}; \;\; w_v = -\frac{1}{2}; \;\; w_{v+1} = 0 \tag{17}$$

and the amplitudes of waves 1 and 3 amount to

$$\begin{aligned} |A_1| &\simeq |1+j\left[c^2/(2\delta^2)\right] \sin \delta z\, e^{j\delta z}| \\ |A_3| &\simeq \left[c^2/(2\delta^2)\right] |\sin \delta z|\,. \end{aligned} \tag{18}$$

According to this approximation only the fraction $c^4/(4\delta^4)$ of the input power converts maximally to wave 3 in the other outer guide. The rest of the input power remains predominantly in wave 1, but to a small extent also in wave 2. However the fraction which wave 1 will maximally loose to waves 2 and 3 under these conditions is only c^2/δ^2 of the input power.

This coupled-wave analysis has been evaluated for duplexer models consisting of three coupled film waveguides [1]. The results have verified the expectations for good wavelength selectivity.

3. Diffused channel guide coupler

More realistically a wavelength-selective directional coupler could consist of three parallel channel-guides diffused into a substrate from diffusion sources in the form of strips of the same configuration as in Fig. 2 on the surface of the substrate. As the strip material diffuses into the substrate it distributes underneath each strip with a relative concentration according to Fig. 3. To this concentration corresponds an index difference to the substrate index n_b with the same relative distribution.

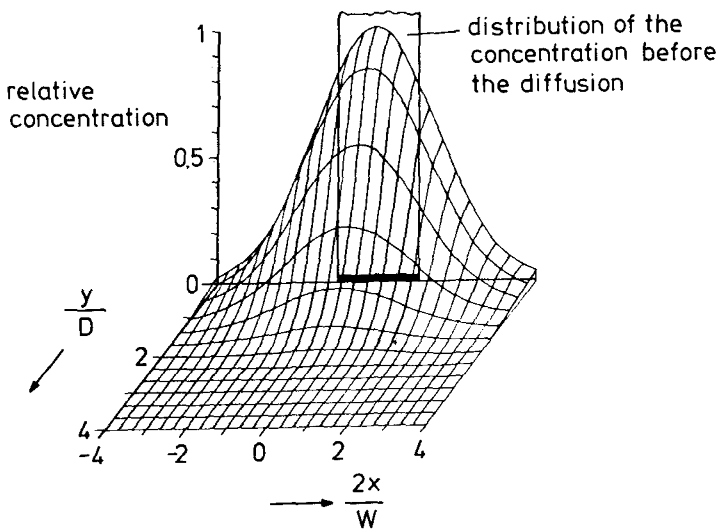

Fig. 3:
Relative Ti-concentration after diffusing a thin Ti-strip into a $LiNbO_3$ substrate (D is the diffusion length) [2]

The three-dimensional wave propagation in three such diffused channel guides running parallel to each other is reduced here to a two-dimensional problem by introducing the effective index for the lowest order mode perpendicular to the substrate surface [3–6]. The wave coupling in this effective-index distribution of the three parallel diffused channel-guides is than analysed with the propagating beam method [7, 8].

4. Index profile and effective index of a diffused channel-guide

Starting from a refractive index n_b in the bulk of the substrate the strip material in Fig. 3 as it diffuses into the substrate will raise the refractive index in the vicinity underneath the strip to

$$n(x,y) = \sqrt{n_b^2 + (n_s^2 - n_b^2) f(y/D) g(2x/W)} \tag{19}$$

with

$$f(y/D) = \exp(-y^2/D^2) \tag{20}$$

and

$$g(2x/W) = \frac{1}{2}\left\{\mathrm{erf}\left[\frac{1}{2}\cdot\frac{W}{D}\left(1+2\frac{x}{W}\right)\right] + \mathrm{erf}\left[\frac{1}{2}\cdot\frac{W}{D}\left(1-2\frac{x}{W}\right)\right]\right\} \tag{21}$$

where W is the width of the strip, D the diffusion length and erf[ ] designates the error-function. n_s is the refractive index that a one-dimensional diffusion would produce at the substrate surface, when the diffusion source covers the entire surface.

At each position x the index profile in Eq. (19) is Gaussian in y direction. For this y-profile we can define a normalized waveguide depth parameter v(x) according to

$$v(x) = kD\left[(n_s^2 - n_b^2)\, g\left(\frac{2x}{W}\right)\right]^{1/2} = v_o\left[g\left(\frac{2x}{W}\right)\right]^{1/2} \tag{22}$$

with $k = 2\pi/\lambda$ as the wavenumber and a normalized mode index b(x) according to

$$b(x) = \frac{n_{eff}^2(x) - n_b^2}{(n_s^2 - n_b^2)\, g\left(2\frac{x}{W}\right)} \tag{23}$$

where $n_{eff}(x)$ then represents the effective index for a particular modal order in the y-profile. At each position x the normalized parameters v(x) and b(x) are related by the following WKB-approximation of the characteristic or dispersion equation

$$\frac{2}{D} v(x) \int_0^{y_t} \left[f(y/D) - b(x)\right]^{1/2} dy = \left(2m + \frac{3}{2}\pi\right) \tag{24}$$

with $m = 0, 1, 2\ldots$

Here y_t is the turning point at which the fields of the particular mode become evanescent in y-direction. y_t is determined by

$$f(y_t/D) = b(x). \tag{25}$$

From Eq. (23) the corresponding mode in the equivalent one-dimensional graded index waveguide has the effective index

$$n_{eff}(x) = \sqrt{n_b^2 + (n_o^2 - n_b^2)\, h(x)} \tag{26}$$

where

$$h(x) = g\left(\frac{2x}{W}\right) b(x)/b_o \tag{27}$$

and

$$b_o = \frac{n_o^2 - n_b^2}{n_s^2 - n_b^2} \tag{28}$$

b_o and n_o are the normalized mode index and the effective index, respectively for the one-dimensional diffusion profile.

The diffused channel-guide of Fig. 3 has the distribution $n'_{eff}(x)$ of effective index for a mode of order m in y-direction. For the actual effective index n'_{eff} of this mode or for its mode index

$$b' = (n'^2_{eff} - n_b^2) / (n_o^2 - n_b^2) \tag{29}$$

we define the waveguide parameter

$$V' = kW\sqrt{n_o^2 - n_b^2} = V_o b_o^{1/2}\, W/D \tag{30}$$

and with it obtain the WKB-approximation of the characteristic or dispersion equation in the following normalized form

$$V' \int_0^{x_t} [h(x) - b']^{1/2}\, dx = (1 + \frac{1}{2})\pi \tag{31}$$

with $l = 0, 1, 2 \ldots$ as the order of the mode in x-direction and x_t as the turning point of the modal fields in this direction, as it follows from $h(x_t) = b$. By solving the dispersion equation (31) for b' the effective index of a mode can be obtained from Eqs. (29) and (28) as

$$n'_{eff} = n_b + (n_s - n_b)\, b_o b'. \tag{32}$$

5. Effective index profile of triple-channel guide coupler

Usually in a triple-guide coupler with wavelength selectivity the two outer guides will be identical but the center guide must differ from both of them in order to achieve selectivity. In case of diffused channel-guides they must be diffused from different strips as diffusion sources. The strips may have different widths or they may differ in thickness or both the width and the thickness may be different. We assume the latter and start with a strip configuration corresponding to Fig. 2. The concentration to which the material from these strip sources diffuses has a distribution n(x,y) of refractive index associated with it, which may be expressed as follwos

$$n^2(x,y) = n_b^2 + (n_{so}^2 - n_b^2)\, f(y/D) g(x). \tag{33}$$

Here f(y/D) is again the same Gaussian profile as in Eq. (20), while

$$g(x) = g_1(2x/W_o) + b_s\, g_2(2x/W_c) + g_3(2x/W_o) \tag{34}$$

with

$$g_1(2x/W_o) = \frac{1}{2}\left\{\mathrm{erf}\left[\frac{W_o}{2D}\left(1 + \frac{2(x+x_o)}{W_o}\right)\right] + \mathrm{erf}\left[\frac{W_o}{2D}\left(1 - \frac{2(x+x_o)}{W_o}\right)\right]\right\},$$

$$g_2(2x/W_c) = \frac{1}{2}\left\{\mathrm{erf}\left[\frac{W_c}{2D}\left(1 + \frac{2x}{W_c}\right)\right] + \mathrm{erf}\left[\frac{W_c}{2D}\left(1 - \frac{2x}{W_c}\right)\right]\right\}, \tag{35}$$

$$g_3(2x/W_o) = \frac{1}{2}\left\{\operatorname{erf}\left[\frac{W_o}{2D}\left(1+\frac{2(x-x_o)}{W_o}\right)\right] + \operatorname{erf}\left[\frac{W_o}{2D}\left(1-\frac{2(x-x_o)}{W_o}\right)\right]\right\}$$

and

$$b_s = (n_{sc}^2 - n_b^2) / (n_{so}^2 - n_b^2), \tag{36}$$

n_{so} and n_{sc} denote the refractive indices that the diffusion of the source material would produce at the substrate surface if one of the side strips or the center strip respectively would cover the entire surface. $x_o = S + (W_o + W_c)/2$ is the spacing of the center of the side strips from $x = 0$.

Just as in Eq. (26) for the single channel-guide, we can now determine an effective index distribution $n_{eff}(x)$ also for the triple channel-guide. With

$$b(x) = \frac{n_{eff}^2(x) - n_b^2}{(n_{so}^2 - n_b^2)\, g(x)} \tag{37}$$

as the normalized mode index for any particular modal order m in y-direction b(x) and consequently $n_{eff}(x)$ are again solutions of the following WKB-approximation of the dispersion equation

$$\frac{2}{D} V(x) \int_0^{y_t} [f(y/D) - b(x)]^{1/2} dy = (2m + \frac{3}{2}\pi) \tag{38}$$

with $m = 0, 1, 2 \ldots$ and y_t from

$$f(y_t/D) = b(x) \tag{39}$$

as the turning point in y-direction for the modal field distribution. The waveguide parameter V(x) in Eq. (38) is defined by

$$V(x) = kD\left[(n_{so}^2 - n_b^2)\, g(x)\right]^{1/2} \tag{40}$$

and may also be expressed by the product

$$V(x) = V_o \sqrt{g(x)} \tag{41}$$

of the waveguide parameter V_o of a diffused film with maximum index n_{so} and the square root of the profile function g(x).

To determine the effective index distribution $n_{eff}(x)$ for a particular modal order m we start with the distribution V(x) for the waveguide parameter according to Eq. (40) and for the y-profile f(y/D) according to Eq. (20) and solve the dispersion equation (38) for the normalized mode index b(x). With its solution and with Eq. (37) the effective index distribution then follows from

$$n_{eff}^2(x) = n_b^2 + (n_{so}^2 - n_b^2)\, g(x)\, b(x). \tag{42}$$

6. Beam propagation method

The beam propagation method starts from a given initial field distribution at the input of a structure and determines the electromagnetic wave beam in the structure by iterative integration of the wave equation [7, 8]. For this integration the method requires that the refractive index of the structure varies locally only little or only gradually.

In case of the triple-channel-guide, in which quasi TE-modes propagate through the structure, the following scalar wave equation may be employed.

$$\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}+\frac{\partial E^2}{\partial z^2}+k^2 n^2(x,y,z)E = o. \tag{43}$$

In it E(x,y,z) is the electric field amplitude, $k=2\pi/\lambda$ the wave number in vacuum, and n(x,y,z) is the refractive-index, which can be expressed as

$$n(x,y,z) = n_b + \Delta n(x,y,z) \tag{44}$$

where n_b is the refractive-index in the bulk of the substrate, and $\Delta n \ll n_b$.

If $\Delta n = 0$, we obtain from Eq. (43) with a two-dimensional Fourier transformation

$$\frac{d^2}{dz^2}\tilde{E}_o(f_x,f_y,z)+(k^2 n_b^2-4\pi^2 f_x^{\,2}-4\pi^2 f_y^{\,2})\,\tilde{E}_o(f_x,f_y,z) = o \tag{45}$$

where $\tilde{E}_o(f_x,f_y,z)$ is the Fourier transform of the field $E_o(x,y,z)$ for $\Delta n = 0$:

$$\begin{aligned}\tilde{E}_o(f_x,f_y,z) &= F\big[E_o(x,y,z)\big]\\ &= \iint\limits_{-\infty}^{\infty} E_o(x,y,z)\,e^{-j2\pi(f_x x+f_y y)}\,dxdy.\end{aligned} \tag{46}$$

Its inverse Fourier transform is given by

$$\begin{aligned}E_o(x,y,z) &= F^{-1}\big[\tilde{E}_o(f_x,f_y,z)\big]\\ &= \iint\limits_{-\infty}^{\infty} \tilde{E}_o(f_x,f_y,z)\,e^{\,j2\pi(f_x x+f_y y)}\,df_x df_y.\end{aligned} \tag{47}$$

Eq. (45) is solved by

$$\tilde{E}_o(f_x,f_y,z) = \tilde{E}_o(f_x,f_y,o)\,e^{-j\,\sqrt{k^2 n_b^2-4\pi^2 f_x^{\,2}-4\pi^2 f_y^{\,2}}\,z}. \tag{48}$$

When we substitute this solution into Eq. (47) we obtain

$$\begin{aligned}E_o(x,y,z) &= F^{-1}\Big[\tilde{E}_o(f_x,f_y,o)\,e^{-j\sqrt{k^2 n_b^2-4\pi^2 f_x^{\,2}-4\pi^2 f_y^{\,2}}\,z}\Big]\\ &= \iint\limits_{-\infty}^{\infty} \tilde{E}_o(f_x,f_y,o)\,e^{-j\sqrt{k^2 n_b^2-4\pi^2 f_x^{\,2}-4\pi^2 f_y^{\,2}}\,z}\cdot e^{j2\pi(f_x x+f_y y)}\,df_x df_y.\end{aligned} \tag{49}$$

This is the exact solution of Eq. (43) for $\Delta n = 0$. To consider the influence of small variations of the refractive-index we introduce a phase factor Q(x,y,z), and express the propagating field as

$$E(x,y,z) = E_o(x,y,z)\,e^{-jQ(x,y,z)} \tag{50}$$

where $E_o(x,y,z)$ is the exact solution of Eq. (43) with $\Delta n = 0$, as expressed by Eq. (49).

On substituting E(x,y,z) from Eq. (50) into Eq. (43) we obtain

$$\begin{aligned}&(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2})\,E_o-2j\,(\frac{\partial Q}{\partial x}\,\frac{\partial}{\partial x}+\frac{\partial Q}{\partial y}\,\frac{\partial}{\partial y}+\frac{\partial Q}{\partial z}\,\frac{\partial}{\partial z})\,E_o\\&-j\,(\frac{\partial^2 Q}{\partial x^2}+\frac{\partial^2 Q}{\partial y^2}+\frac{\partial^2 Q}{\partial z^2})\,E_o-[(\frac{\partial Q}{\partial x})^2+(\frac{\partial Q}{\partial y})^2+(\frac{\partial Q}{\partial z})^2]\,E_o\\&+k^2n^2(x,y,z)\,E_o=o.\end{aligned} \tag{51}$$

With

$$(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2})\,E_o=-k^2n_b^2E_o \tag{52}$$

and since Q(x,y,z) varies only so gradually in z-direction and even less in x- and y-direction that

$$\frac{\partial^2 Q}{\partial x^2}+\frac{\partial^2 Q}{\partial y^2}+\frac{\partial^2 Q}{\partial z^2}\simeq o, \tag{53}$$

$$(\frac{\partial Q}{\partial x})^2+(\frac{\partial Q}{\partial y})^2\ll(\frac{\partial Q}{\partial z})^2, \tag{54}$$

$$|(\frac{\partial Q}{\partial x}\,\frac{\partial}{\partial x}+\frac{\partial Q}{\partial y}\,\frac{\partial}{\partial y})\,E_o|\ll|\frac{\partial Q}{\partial z}\,\frac{\partial}{\partial z}E_o|, \tag{55}$$

we may approximate Eq. (51) by

$$E_o\,(\frac{\partial Q}{\partial z})^2+2j\frac{\partial E_o}{\partial z}\,\frac{\partial Q}{\partial z}-k^2\left[n^2(x,y,z)-n_b^2\right]E_o=o. \tag{56}$$

To further simplify this differential equation we use the paraxial approximation

$$\frac{\partial E_o}{\partial z}=-jkn_bE_o \tag{57}$$

for $E_o(x,y,z)$ and neglect the term $E_o(\frac{\partial Q}{\partial z})^2$ since it is only of second order in the small quantity $\partial Q/\partial z$. We thus arrive at the following first order differential equation for the phasefactor

$$2kn_b\frac{\partial Q}{\partial z}-k^2\left[n^2(x,y,z)-n_b^2\right]=o \tag{58}$$

which is readily integrated, and yields

$$Q=\frac{kn_b}{2}\int_o^Z\left[\frac{n^2(x,y,z')}{n_b^2}-1\right]dz'. \tag{59}$$

Errors that any one of these approximations may cause, in particular also Eq. (57), are kept small by restricting the integration in Eq. (59) to short intervals Δz of z [9].

From Eqs. (50), (49) and (59) we thus obtain $E(x,y,z+\Delta z)$ from E(x,y,z) for short propagation intervals Δz

$$E(x,y,z+\Delta z)=F^{-1}\left[\tilde{E}_o(f_x,f_y,z)\,e^{-j\sqrt{k^2n_b^2-4\pi^2f_x^2-4\pi^2f_y^2}\Delta z}\right]\cdot e^{-jQ(x,y,z+\Delta z)} \tag{60}$$

where

$$Q(x,y,z+\Delta z) = \frac{1}{2} kn_b \int_{z}^{z+\Delta z} \left[\frac{n^2(x,y,z')}{n_b^2} - 1\right] dz'$$
$$\simeq kn_b \int_{z}^{z+\Delta z} \left[\frac{n(x,y,z')}{n_b} - 1\right] dz'. \tag{61}$$

The inverse Fourier transform in Eq. (60) describes propagation of an electromagnetic wave through a homogeneous medium with refractive-index n_b while the remaining factor introduces a phase change equivalent to propagation through a thin lens.

When we apply this beam propagation method to the two-dimensional effective index distribution of the triple-channel-guide coupler according to Eq. (42), we obtain instead of Eq. (60) the somewhat simpler relation

$$E(x,z+\Delta z) = F^{-1}\left[\tilde{E}_o(f_x,z)\, e^{-j\sqrt{k^2 n_b^2 - 4\pi^2 f_x^2}\,\Delta z}\right] \cdot e^{-jQ(x,\Delta z)}. \tag{62}$$

For short enough intervals Δz we may furthermore approximate the integral of Eq. (61) by:

$$Q(x,\Delta z) = kn_b \left[\frac{n_{eff}(x)}{n_b} - 1\right] \Delta z. \tag{63}$$

6. Solution in terms of a discrete Fourier transform

For the triple-channel coupler we can always define a region $0 < |x| < X_o$ outside of which the fields remain small enough to be neglected. We can then let the field distribution repeat periodically outside this region

$$E(x+mX_o,z) = E(x,z) \qquad m = 0, \pm 1, \pm 2, \ldots \tag{64}$$

If this field distribution has only a limited number of spectral components, then there will be an one-to-one correspondence between the elements of its discrete Fourier transform and the Fourier transform coefficients, provided the sampling rate is at least twice the highest frequency [10]. Fig. 4 shows this correspondence by comparing a continuous wave form E(x) and its Fourier transform $\acute{E}(f_x)$ in Fig. 4a with a discrete series $E(l\Delta x)$, which is formed from the samples of E(x), and its discrete Fourier transform $\tilde{E}(n\Delta f_x)$ in Fig. 4b.

From the sampling values $E_l(z)$ of the field the discrete Fourier transform can be calculated according to

$$\tilde{E}_n(z) = \sum_{l=o}^{N-1} E_l(z) e^{-j2\pi n \frac{l}{N}} \tag{65}$$

The inverse discrete Fourier transform then follows as

$$E_l(z) = \frac{1}{N} \sum_{n=o}^{N-1} \tilde{E}_n(z) e^{j2\pi n \frac{l}{N}}, \tag{66}$$

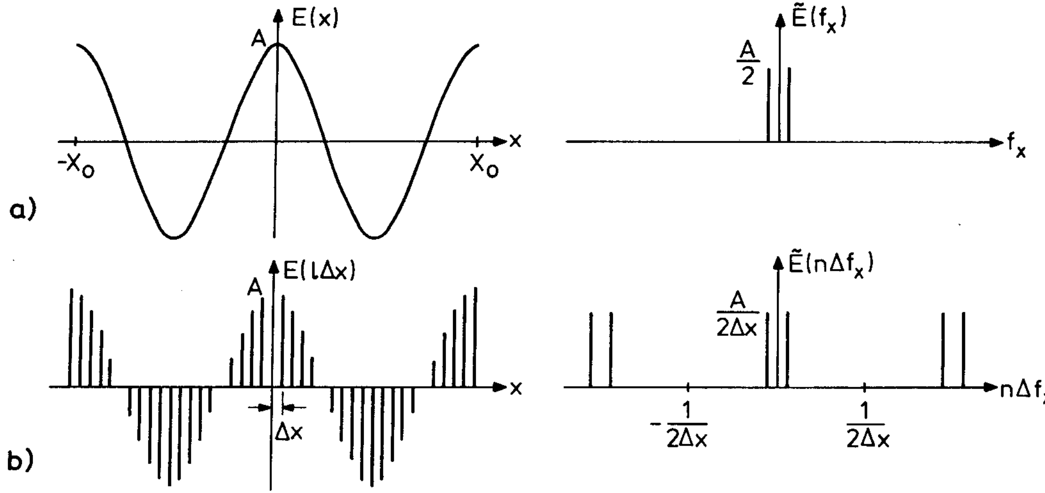

Fig. 4:
a) Continuous wave function and its Fourier transform
b) Discrete series and its discrete Fourier transformation

with

$$E_l(z) = E(l\Delta x, z), \tag{67}$$

$$\tilde{E}_n(z) = \tilde{E}(n\Delta f_x, z), \tag{68}$$

$$\Delta x = X_o/N, \tag{69}$$

$$\Delta f_x = 1/X_o. \tag{70}$$

The number N of sampling points will be determined by X_o and θ_{max}

$$\frac{N\pi}{X_o} = |k_{x\,max}| > k \sin\theta_{max}, \tag{71}$$

where

$$\theta_{max} \simeq (n_{max}^2 - n_b^2)^{1/2} \tag{72}$$

is the largest possible value of the angle θ between the direction of a representative plane wave with transverse wavevector k_x and the z axis

$$\theta = \arcsin(k_x/k). \tag{73}$$

If a periodic repetition of E(x,z) is assumed, the Fourier transform in Eq. (62) will be replaced by the discrete Fourier transform

$$E_l(z+\Delta z) = F_D^{-1}\left[\tilde{E}_n(z) e^{-j\sqrt{k^2 n_b^2 - k_x^2}\Delta z}\right] e^{-jQ(x,\Delta z)} \tag{74}$$

where $Q(x,\Delta z)$ is still as in Fig. (63), and

$$k_x = \begin{cases} 2\pi(n-1)/X_o & \frac{N}{2}+1 \geq n \geq 1, \\ 2\pi(n-1-N)/X_o & N > n > \frac{N}{2}+1. \end{cases} \tag{75}$$

7. Normal modes of the triple-channel-guide

The triple waveguide structure of the wavelength selective directional coupler guides a finite number of normal modes, of which at least three are excited, when power transfers from one outer guide to the other. To determine the amplitude w_n and phase coefficients β_n of these normal modes, the field amplitude correlation function

$$p(z) = \int E^*(x,o) \cdot E(x,z) dz \tag{76}$$

is subjected to the Fourier transformation [11]

$$p(\beta) = \int p(z)\, e^{-j\beta z} dz. \tag{77}$$

Since p(z) can also be expressed as

$$p(z) = \sum_n |w_n|^2 e^{-j\beta_n z} \tag{78}$$

its Fourier transform for $z \to \infty$ is

$$p(\beta) = \sum_n |w_n|^2 \delta(\beta - \beta_n). \tag{79}$$

For the finite length $z=Z$ of an actual coupler the peaks of $p(\beta)$ occur at the β_n and indicate with their height the power in the respective normal mode. To reduce the uncertainty with which the β_n-values can only be determined, a window-function $w(z)$ is used to weigh $p(z)$ before its Fourier transform is computed. The resonances of $p(\beta)$ will then be shaped by this window function.

The maximum uncertainty in β_n is

$$\Delta\beta_n = \frac{1}{2}\Delta\beta = \pi/Z \tag{80}$$

where $\Delta\beta$ is the sampling interval in β.

The uncertainty in β_n is now reduced by fitting the correct line-shape function intrinsic to $w(z)$ to the sampled values of $p(\beta)$ that are closest to the resonance under consideration and by determining the β_n from the resulting line-shape fit.

For the Hanning window

$$w(z) = 1\text{-}\cos\frac{2\pi z}{Z}, \tag{81}$$

β_n can be calculated from [12]

$$\beta_n = \beta'_m - \frac{2\pi\delta'}{Z} \tag{82}$$

where $\beta'_m = m'\Delta\beta$ is the local maximum in the sampled values of $p(\beta)$, and

$$\delta' = \begin{cases} \dfrac{3r+(9r^2-8)^{1/2}}{2} & r<0 \\ \dfrac{3r-(9r^2-8)^{1/2}}{2} & r>0, \end{cases} \tag{83}$$

with

$$r = \frac{1+R}{1-R}, \tag{84}$$

and

$$R = \frac{p[(m'+1)\Delta\beta]}{p[(m'-1)\Delta\beta]}. \tag{85}$$

The normalized line-shape function for the Hanning window according to Eq. (81) is defined by

$$L(\beta-\beta_n) = \frac{1}{Z}\int_0^Z e^{j(\beta-\beta_n)z}\cdot w(z)dz. \tag{86}$$

After the normal mode phase coefficients corresponding to $\beta_{v-1}, \beta_v, \beta_{v+1}$ of the simple coupled-wave theory in Eqs. (15) have been determined by this spectral analysis, the requirement for complete power transfer namely

$$\beta_{v-1} - \beta_v = \beta_v - \beta_{v+1} \tag{87}$$

may be checked, and if its satisfied, the length of the coupler for complete power transfer may be determined from Eq. (14) or

$$L_c = \pi/(\beta_{v-1} - \beta_v). \tag{88}$$

8. Coupler design

From the analysis of the film waveguide model for the triple waveguide coupler [13] we have learned that high wavelength selectivity will be achieved when not the fundamental mode of the center waveguide but its next higher order mode serves to transfer power between the outer two guides. The center guide should therefore be designed to be phase matched with this higher order mode to the fundamental mode of the outer waveguides at the wavelength at which complete power transfer is desired. For loose coupling between these modes this condition for phase match will lead to normal modes of the composite structure that satisfy Eq. (87). If the guides, are more tightly coupled, however, this design must be modified in order to meet Eq. (87). In designing the triple waveguide coupler we have nevertheless started by phase matching the respective modes of the isolated channel-guides first and for tighter coupling have made corrections subsequently.

In the triple waveguide coupler the center waveguide extends only over the coupling length L_c while the outer waveguides continue beyond this length to connect with other components. For the beam propagation analysis the input channel was made long enough so that from its point of excitation by a Gaussian field distribution to where it begins to couple to the center guide the field distribution of its fundamental mode is well established. If the initial Gaussian field distribution is well matched in its spot size to the fundamental mode of the channel-guide the excitation efficiency is better than 99%, typically even 99.4%.

The periodic repetition in the transverse x-direction of the initial field distribution, which is assumed here for the discretisation of the Fourier transforms, acts as if the boundaries on both sides of the coupler reflect the waves as they spread sideways. When these reflections interfere with the primary wave they lead to parasitic modes in the normal mode spectrum as it obtains from the Fourier transform (77) of the field amplitude correlation function (76). To eliminate these parasitic modes we suppress the reflections by placing absorbers at the boundaries. In the examples to follow the substrate was taken to be 100 μm wide, but absorbers were placed on both sides only 30 μm from the center line of the coupler. This narrow spacing of absorbers reduces the computation time considerably. The computational steps for the beam propagating method in Eq. (63) were chosen at $\Delta z = 10$ μm for all numerical examples.

9. Examples

As a first example for complete conversion at $\lambda = 1.5$ μm we let the coupling between the guides be only loose by choosing the spacing between source strips relatively wide

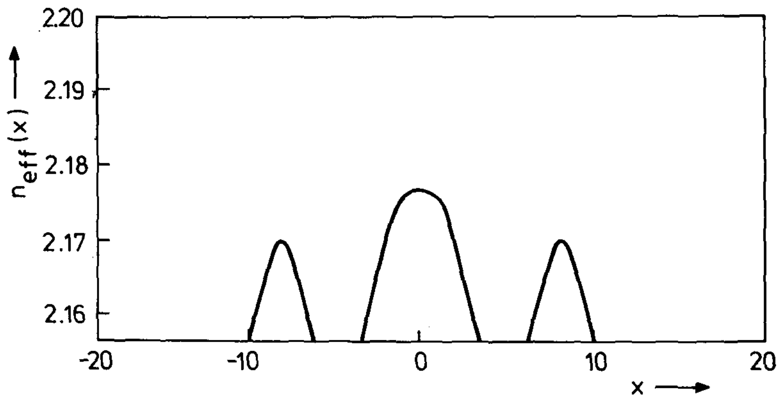

Fig. 5:
Effective index distribution of a triple channel guide coupler with
$W_o = 3\ \mu m$, $W_c = 6.1\ \mu m$, $S = 4\ \mu m$, $n_b = 2.1567$, $n_{so} = n_{sc} = 2.2262$ at $\lambda = 1.5\ \mu m$

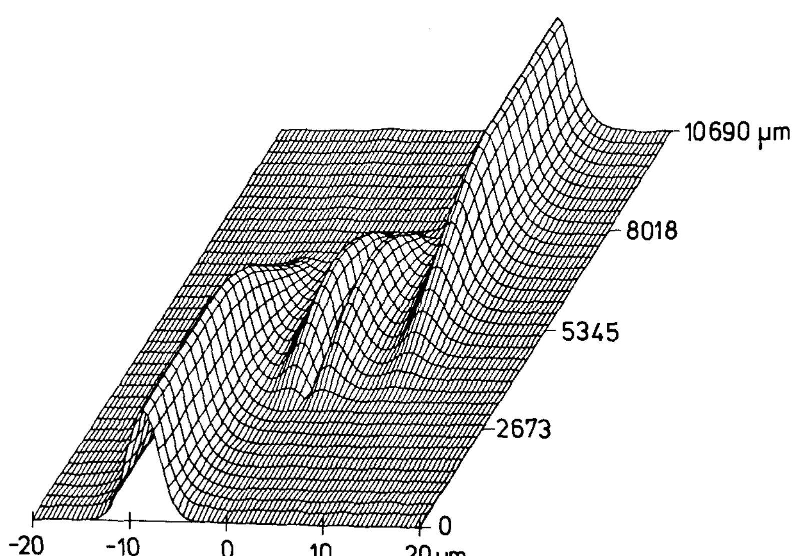

Fig. 6:
Field amplitude in the triple-channel guide coupler of Fig. 5 at the wave length 1.5 μm for complete power conversion

at $S = 4$ μm. A coupler design based on phase matching of the isolated channel-guide modes is listed in Fig. 5, and at $\lambda = 1.5$ μm has the effective index-distribution of this figure.

The amplitude distribution of the field for complete power conversion in this coupler is shown Fig. 6. One of the outer waveguide arms is excited by a Gaussian distribution at the wavelength at which Eq. (88) is satisfied. Power enters the coupler therefore only in the fundamental mode of this outer guide. As the field distribution propagates through the coupling region it converts its power to the first order mode of the center guide from where it transfers to the fundamental mode of the other outer guide.

The amplitudes w_n and phase-coefficients β_n of the normal modes of the composite structure, as they are excited under these conditions, are determined by computing first the field amplitude correlation function according to Eq. (76) and subjecting it to the Fourier transformation (77). Fig. 7 shows the normal mode spectrum $p(\beta)$ that corresponds to Fig. 6. Only three normal modes are excited. Their phase coefficients satisfy the condition (87) for complete power transfer and the coupling length L_c is related to the difference in normal mode phase-coefficients by Eq. (88).

At this length of the coupler the beam propagation analysis yields fractional power in each of the three waveguides as Fig. 8 shows it as a function of wavelength. At wavelengths shorter than $\lambda = 1.44$ μm, in the present example, the coupler transmits straight through the respective outer guide with only very little power conversion loss. Fig. 9 shows the normal mode spectrum for a wavelength in the range of straight-through transmission, namely at $\lambda = 1.3$ μm. It has only one peak and indicates that the two normal normal modes of the composite structure which, according to Eq. (17), are predominantly and with equal amplitude excited under these conditions are nearly degenerate with respect to their phase-coefficients according to Eq. (15).

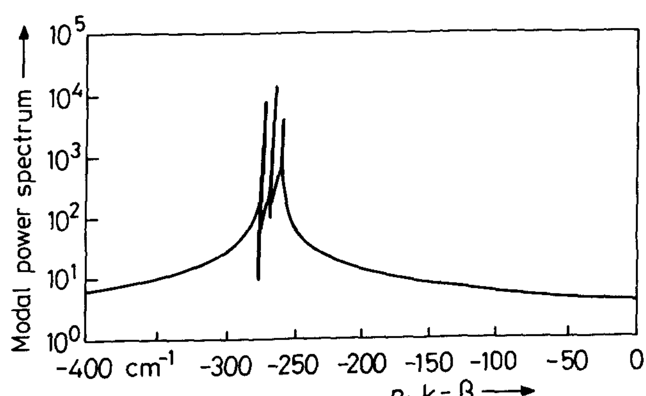

Fig. 7

Normal mode spectrum of the triple channel guide coupler of Fig. 5 at the wavelength of complete power conversion

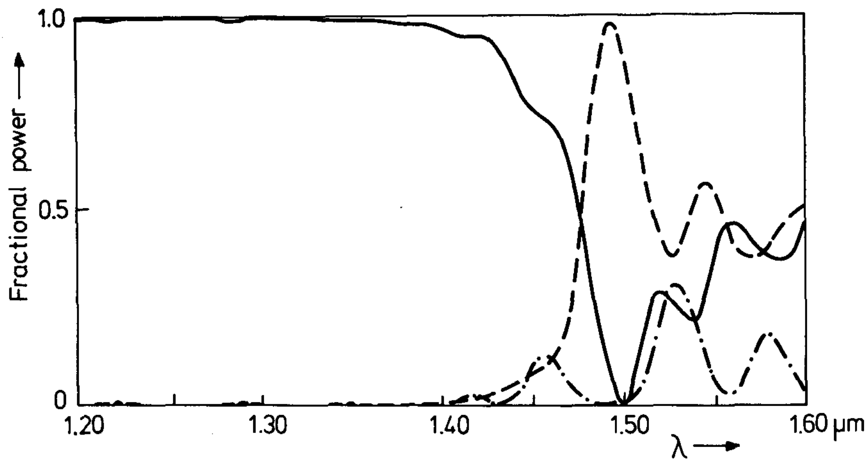

Fig. 8:
Fractional power in each of the three channels after propagation through the coupling length L_c in the coupler of Fig. 5

——— through going outer guide
— — — coupled outer guide
—·—·— center guide

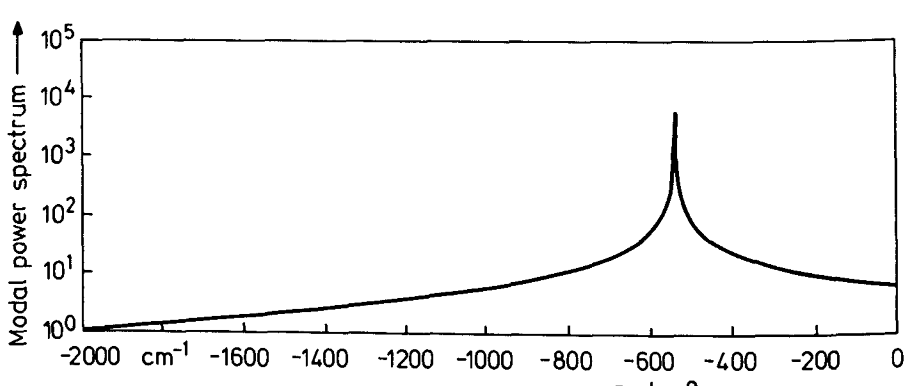

Fig. 9:
Normal mode spectrum of the triple channel guide coupler of Fig. 5 for straight through transmission at $\lambda = 1.3\ \mu m$

At any wavelengths longer than the wavelength $\lambda = 1.5$ μm for complete power conversion the power out of the coupler appears divided among the three waveguides. In this spectral range and at even longer wavelengths phase discrimination between the interacting modes of the individual channel-guides remains too small and coupling between these modes becomes too strong to prevent power conversion from one of the outer guides to the center guide.

Fig. 10 shows the spectral characteristics of power conversion for less spacing between the channel-guides. The coupling between the modes of the individual channel-guides is much tighter in this case so that the source strip for the diffusion of the center guide had to be adjusted in its yield to raise the refractive index distribution of this guide to such a level, that the three dominant normal modes of the composite structure satisfy the requirement of Eq. (87) for complete power transfer again at $\lambda = 1.5$ μm. Fig. 11 shows the modal amplitude spectrum of the composite structure from Eq. (76) and (77) as it is excited in the case of complete power transfer at $\lambda = 1.5$ μm. The three predominantly excited normal modes have indeed equal spacing and the mode with the intermediate phase coefficient has twice the amplitude, just as it is predicted by the coupled wave analysis of the three-mode model. Note however that the small peak in the modal amplitude spectrum at the larger phase coefficient indicates the excitation of a lower order mode of the composite structure which acts as a parasite and degrades the coupler performance.

Because of tighter coupling between the modes of the individual channel-guides the three dominant normal modes of the composite structure in Fig. 11 differ more in

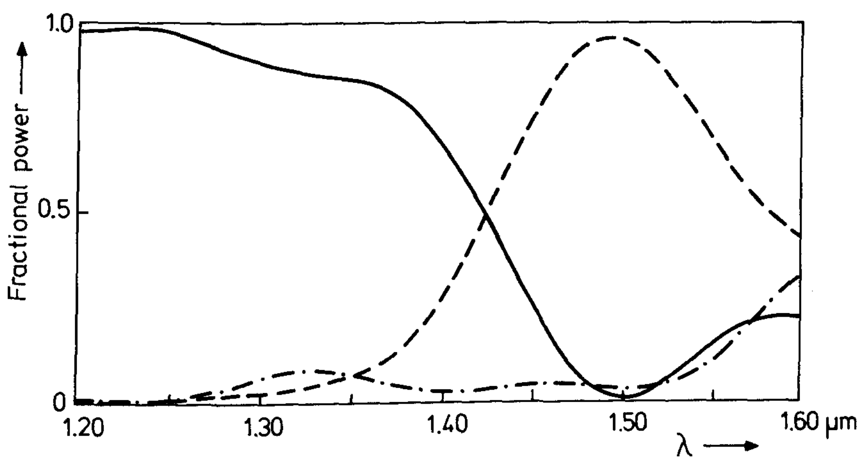

Fig. 10

Fractional power in each of the three channels with less spacing between channels than in Fig. 5–9
$n_b = 2.1567$, $n_{so} = 2.2262$, $n_{sc} = 2.2265$, $W_o = 3$ μm, $W_c = 6.1$ μm, $S = 2$ μm and $L_c = 1.03$ mm

——————— *through going outer guide*

— — — — — — *coupled outer guide*

— · — · — · — *center guide*

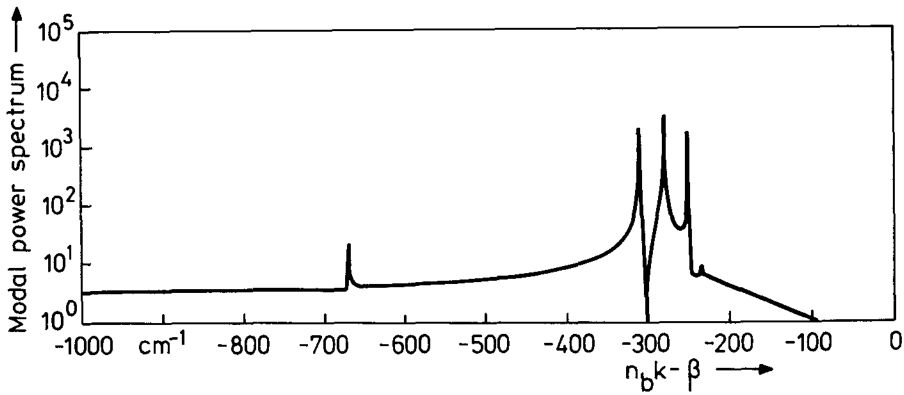

Fig. 11:
Normal mode spectrum of the triple channel guide coupler in Fig. 10 at the wavelength λ = 1.5 μm for complete power conversion

phase coefficient than in Fig. 9 for wider spacing between channels and less coupling as a consequence. Also the coupling length from Eq. (88) is shorter and the selectivity lower.

For straight-through transmission in this tightly coupled structure at $\lambda = 1.3$ μm the normal mode spectrum in Fig. 12 shows that the two predominantly excited modes are nearly degenerate in phase coefficients as Eqs. (15) indicate and that, in accordance with Eqs. (17), the mode $\nu+1$ is not excited.

For channel-guide spacings which are in between the spacings of these two examples, and also for less spacing between the channel-guides, the coupling length for complete power transfer at $\lambda = 1.5$ μm depends on the spacing S between the source strips for the diffused channel-guides as Fig. 13 shows it. Also plotted versus the strip spacing is in this figure the 1-dB bandwidth of power conversion to the other outer guide, as well as the insertion loss at $\lambda = 1.3$ μm through each of the outer channel-guides. Note that for somewhat less than 2 μm strip spacing the coupling length for $\lambda = 1.5$ μm remains shorter than 1 mm, the 1-dB bandwidth of power-conversion is nearly $\Delta\lambda = 100$ nm while only little more than 0.5 dB of power-conversion loss must be tolerated at $\lambda = 1.3$ μm.

The coupling-length can also be reduced and the power-conversion band widened, when the channel-guides have each less index difference with respect to the bulk of the substrate and more coupling between them as a consequence. The power-conversion characteristics in Fig. 14 for such a coupler with less index difference has nearly the same form for 3 μm strip spacing as the previous examples with the then larger index difference have them for 2.6 μm strip spacing and the same coupling length.

When fabricating a triple-waveguide coupler to perform in accordance with given specifications its dimensions and index profiles must stay within certain tolerances. Fig. 15 shows the additional loss at $\lambda = 1.5$ μm when the triple waveguide coupler for

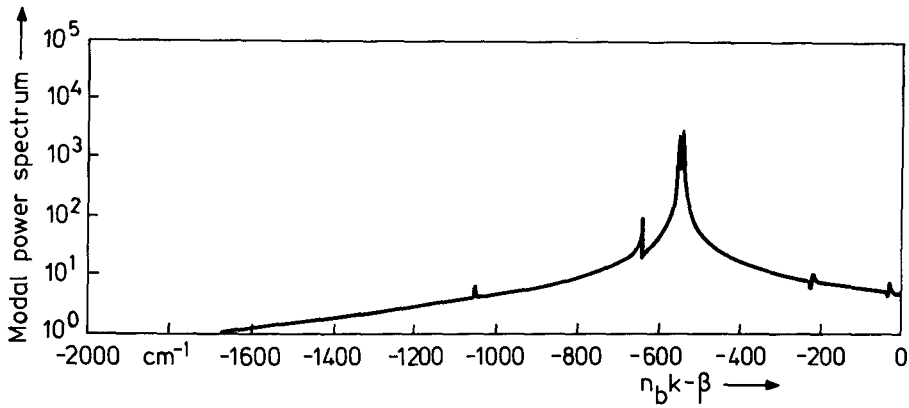

Fig. 12:
Normal mode spectrum of the triple channel guide coupler in Fig. 10 at the wavelength λ = 1.3 μm for straight through transmission

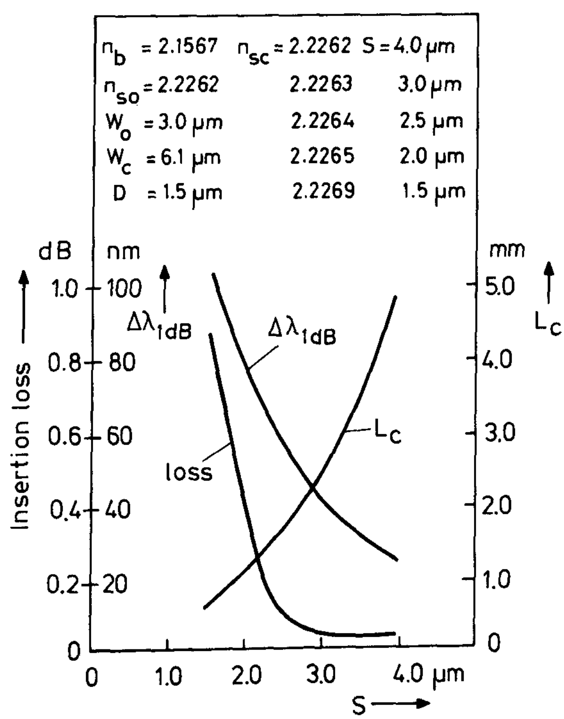

Fig. 13:
Coupling length L_c and 1-dB bandwidth Δλ at λ = 1.5 μm and insertion loss at λ = 1.3 μm of triple-channel couplers

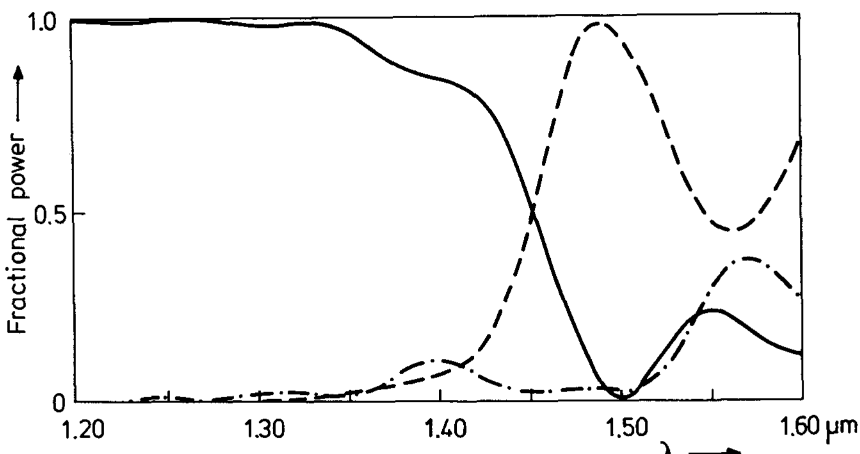

Fig. 14:
Power conversion characteristic for triple-channel coupler with channels of less index difference
——————— *through going outer guide*
— — — — — — *coupled outer guide*
—·—·—·—·— *center guide*

complete power-conversion at $\lambda = 1.5$ μm deviates in a number of ways from its correct design. The index-difference

$$\Delta_c = (n_{sc}^2 - n_b^2) / (2n_{sc}^2)$$

of the center guide appears to be the most critical parameter. When this index difference deviates only by 0.7% from its correct value complete power-conversion from one to the other outer guide suffers already 1 dB of additional loss. One dB of additional loss is also incurred when the index difference Δ_o of the outer guides deviates by 0.9% from its nominal value, while the width of the source strip for the center channel may deviate by 1% and the width of the source strip for the outer guide by 1.4% from their respective nominal values before the additional loss reaches 1 dB. The spacing S of the source strips may even deviate by 7% from the nominal value before 1 dB of additional loss is incurred. The fabrication has to be correspondingly accurate or means must be found to correct for any deviations.

Conclusions

Triple waveguide directional couplers in the form of three parallel diffused channel-guides have sufficient wavelength selectivity to be employed as wavelength multiplexers or duplexers in monomode systems. Their short coupling length suits optical integration while the wide band for complete power-conversion relaxes wavelength stability. Coupled mode analysis and beam propagation method facilitate the design of these couplers.

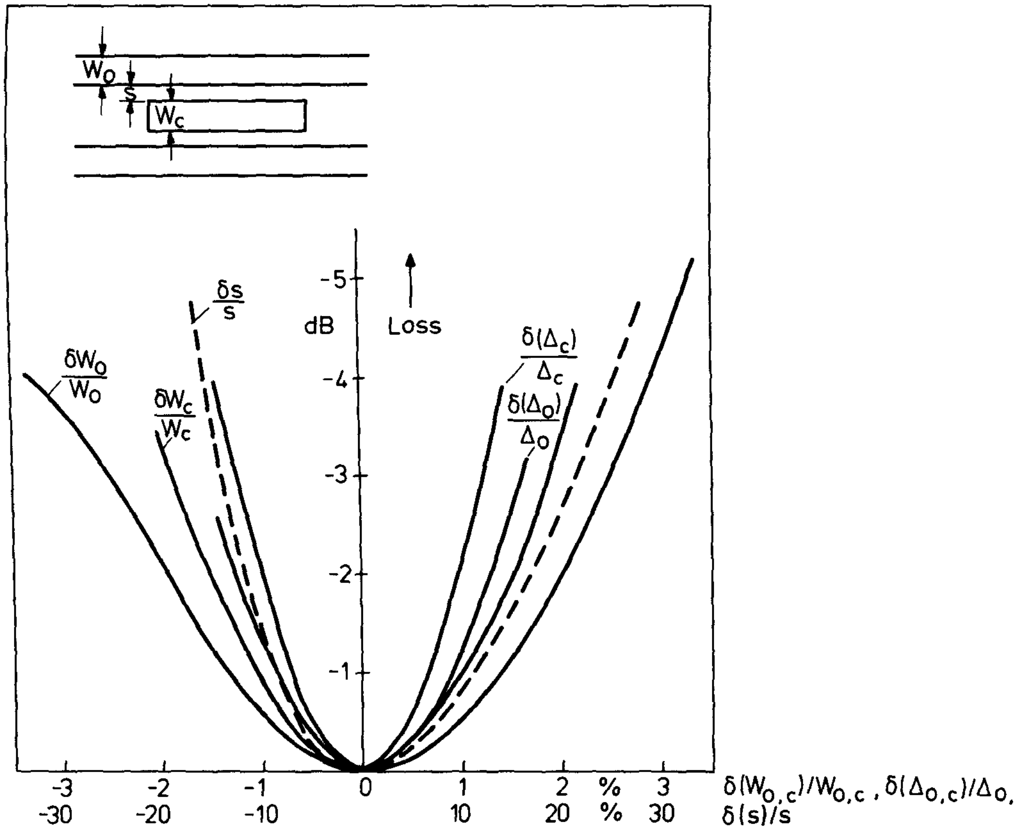

Fig. 15:
Additional loss of complete conversion at $\lambda = 1.5$ μm in a triple channel guide coupler due to deviation of coupler parameters from their nominal values

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